49

18 p.

N64-27272

~~X64 30124~~

Code 2A

NASA TMX 51819

Cat. 22

MTP-ASTR-N-63-9
August 28, 1963

GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

DYNAMIC SIMULATION OF ST-124 PLATFORM

By

C. D. Carlile

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

GEORGE C. MARSHALL SPACE FLIGHT CENTER

MTP-ASTR-N-63-9

DYNAMIC SIMULATION OF ST-124 PLATFORM

By

C. D. Carlile

ABSTRACT

To evaluate the guidance and control loop of the Saturn vehicle before the vehicle is flown, it is necessary to simulate each portion of the hardware. These simulations are then assembled as part of a closed loop simulation of the overall vehicle.

This report covers the portion which has to do with the ST-124 stabilized platform. The ST-124 has four gimbals.

Equations are derived that permit the individual gimbal angles of the ST-124 to be studied. These equations are checked by comparing with more conventional equations. The applications of the derived equations as inputs for the guidance and control loop for Saturn are discussed.

GEORGE C. MARSHALL SPACE FLIGHT CENTER

MTP-ASTR-N-63-9

DYNAMIC SIMULATION OF ST-124 PLATFORM

By

C. D. Carlile

TABLE OF CONTENTS

DEFINITION OF SYMBOLS

X_s, Y_s, Z_s	Space-fixed coordinate system for guidance computation.
X_m, Y_m, Z_m	Vehicle-fixed coordinate system for computation of dynamics.
t	Time from launch.
t_o	Time of launch.
χ_y, χ_x, χ_z	Eulerian angles defining desired orientation of vehicle-fixed coordinate system in space-fixed guidance coordinate system (X_s, Y_s, Z_s) when executed in the sequence χ_y, χ_x, and χ_z about the vehicle-fixed axis indicated with subscripts. The vehicle-fixed coordinate system is parallel with the guidance coordinate at launch.
θ_r	Inner gimbal angle (roll).
θ_{pl}	Inner-middle gimbal angle (pitch limited).
θ_y	Outer-middle gimbal angle (yaw).
θ_{op}	Outer gimbal angle (outer pitch).
$\dot{\theta}_r$, $\dot{\theta}_{pl}$, $\dot{\theta}_y$, $\dot{\theta}_{op}$	Scalar components of the vehicle's angular velocity along the platform gimbal axes.
$\dot{\phi}_x$, $\dot{\phi}_y$, $\dot{\phi}_z$	Scalar components of the vehicle's angular velocity along the vehicle yaw, roll, and pitch axes, respectively.
ψ_p, ψ_r, ψ_y	Instantaneous error signals in pitch, roll, and yaw, respectively, obtained from resolver chain.
β_p, β_y, β_r	Actual gimbaled engine angular position for pitch, yaw, and roll, respectively.
β_{pc}, β_{yc}, β_{rc}	Gimbaled engine angular position for pitch, yaw, and roll, respectively, as computed by control computer.

DEFINITION OF SYMBOLS (Continued)

GSP	The guidance signal processor is a modular package which provides flexibility in the Saturn I guidance and control loop. It contains the attitude command resolver chain as well as other components which cannot be integrated into the guidance computer because of mission diversification and design alteration.

GEORGE C. MARSHALL SPACE FLIGHT CENTER

MTP-ASTR-N-63-9

DYNAMIC SIMULATION OF ST-124 PLATFORM

By

C. D. Carlile

SUMMARY

To evaluate the guidance and control loop of the Saturn vehicle before the vehicle is flown, it is necessary to simulate each portion of the hardware. These simulations are then assembled as part of a closed loop simulation of the overall vehicle.

This report covers the portion which has to do with the ST-124 stabilized platform. The ST-124 has four gimbals.

Equations are derived that permit the individual gimbal angles of the ST-124 to be studied. These equations are checked by comparison with more conventional equations. Finally, the applications of the derived equations as inputs for the guidance and control loop for Saturn are discussed.

SECTION I. INTRODUCTION

To provide the highest degree of confidence in the Saturn vehicle's guidance and control loop, it is necessary to simulate each portion of the hardware. These simulations are then assembled as part of a closed loop simulation of the overall vehicle. Error tolerances are then introduced and the effect on the mission is analyzed.

The three guidance commands (χ_x, χ_y, χ_z) are taken from the output of the guidance computer. The resolvers of the guidance signal processor (GSP) are simulated by matrix transformation. The resolver outputs of the GSP are routed to the inputs of the platform resolver chain. The dual frequency characteristics of the GSP resolver chain and platform resolver chain are represented by two identical matrix transformations with different inputs.

The platform resolver chain is represented by matrix transformations as usual. Equations are derived to compute the ST-124 gimbal angles (θ_r, θ_{pl}, θ_y) for this transformation. θ_{op} equals X_z, a known angle.

The appropriate outputs of the platform resolver chain are routed to the control computer. Engine swivel angles are computed in the control computer. These swivel angles are inputs to equations of motion which represent the vehicle.

SECTION II. ST-124 GIMBAL ANGLES

The equations of motion of a rigid body, such as a rocket in flight, require a set of equations to compute the attitude of the vehicle relative to a space-fixed coordinate system. A conventional set of these equations is stated in the Appendix.

The gimbal angles of a stabilized platform define the attitude of the vehicle relative to the space-fixed coordinate system maintained by the platform. The gimbal angles of the ST-124 stabilized platform are important quantities in the operation of the guidance and control system.

Coordinate transformation resolvers are used in each gimbal to transform guidance commands from space-fixed coordinates to vehicle coordinates. A detailed study of the resolver chain in the guidance and control loop requires a knowledge of the gimbal angles. The design of the ST-124 stabilized platform makes the proper functioning of the guidance and control system dependent upon the relative magnitudes of the platform gimbal angles. The magnitude of the gimbal angles is dependent upon the guidance commands and the dynamic response of the vehicle to these commands; therefore, it is extremely desirable to simulate the platform gimbal angles in the guidance and control loop of the vehicle simulation.

SECTION III. ST-124 GIMBAL ANGLE EQUATIONS

The vehicle's angular velocity is represented by a vector, $\vec{\omega}$, perpendicular to the plane of the rotation angle. The gimbal angle velocities are $\dot{\theta}_r$, $\dot{\theta}_{pl}$, $\dot{\theta}_y$, and $\dot{\theta}_{op}$. These are scalar components of the vehicle's angular velocity vector along the platform gimbal axes.

If it is assumed that $\dot{\theta}_r$, $\dot{\theta}_{pl}$, $\dot{\theta}_y$, $\dot{\theta}_{op}$, and the gimbal angles, θ_r , θ_{pl}, θ_y, θ_{op}, are known; the scalar components of the vehicle angular velocity vector $\dot{\phi}_x$, $\dot{\phi}_y$, $\dot{\phi}_z$ along the vehicle axes may be computed by considering the transformation instrumented by the platform.

$$\begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} = A_4\ A_3\ A_2\ A_1 \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} \qquad (1)$$

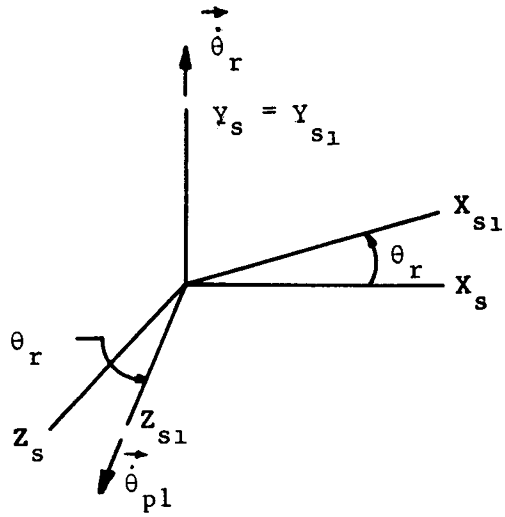
$$A_1 = \begin{bmatrix} \cos\theta_r & 0 & -\sin\theta_r \\ 0 & 1 & 0 \\ \sin\theta_r & 0 & \cos\theta_r \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos\theta_{p1} & \sin\theta_{p1} & 0 \\ -\sin\theta_{p1} & \cos\theta_{p1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_y & \sin\theta_y \\ 0 & -\sin\theta_y & \cos\theta_y \end{bmatrix}$$

$$A_4 = \begin{bmatrix} \cos\theta_{op} & \sin\theta_{op} & 0 \\ -\sin\theta_{op} & \cos\theta_{op} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

θ_r is a rotation about the Y_s axis; $\dot{\theta}_r$ is along the Y_s axis.



$\dot{\theta}_r$ is not affected by this rotation, but it will be affected by the three succeeding rotations. The components of $\dot{\phi}_x$, $\dot{\phi}_y$, and $\dot{\phi}_z$ as a result of θ_r are computed by

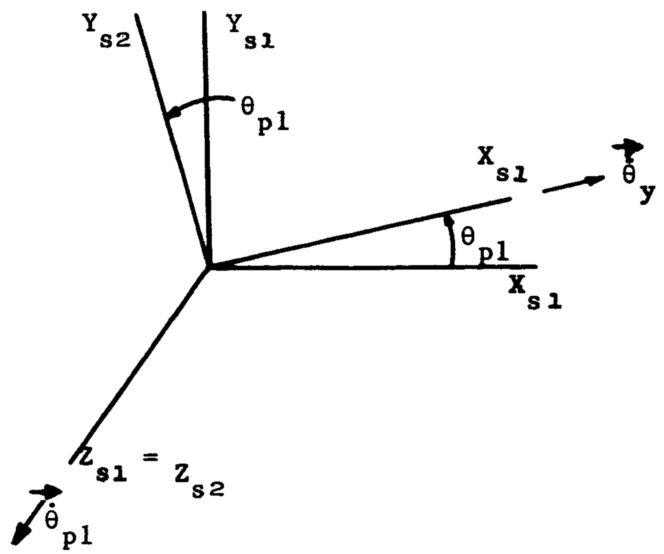
$$\begin{bmatrix} \dot{\phi}_{x\dot{\theta}_r} \\ \dot{\phi}_{y\dot{\theta}_r} \\ \dot{\phi}_{Z\dot{\theta}_r} \end{bmatrix} = A_4 \; A_3 \; A_2 \begin{bmatrix} 0 \\ \dot{\theta}_r \\ 0 \end{bmatrix} \tag{2}$$

A_2, A_3, and A_4 are rotation matrices defined above.

$\dot{\theta}_{p1}$ is in the X_{s1}, Y_{s1}, Z_{s1} coordinate system; $\dot{\theta}_{p1}$ is along Z_{s1}. The components of $\dot{\phi}_x$, $\dot{\phi}_y$, and $\dot{\phi}_z$ as a result of θ_{p1} are computed by

$$\begin{bmatrix} \dot{\phi}_{x\dot{\theta}_{p1}} \\ \dot{\phi}_{y\dot{\theta}_{p1}} \\ \dot{\phi}_{Z\dot{\theta}_{p1}} \end{bmatrix} = A_4 \; A_3 \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{p1} \end{bmatrix} \tag{3}$$

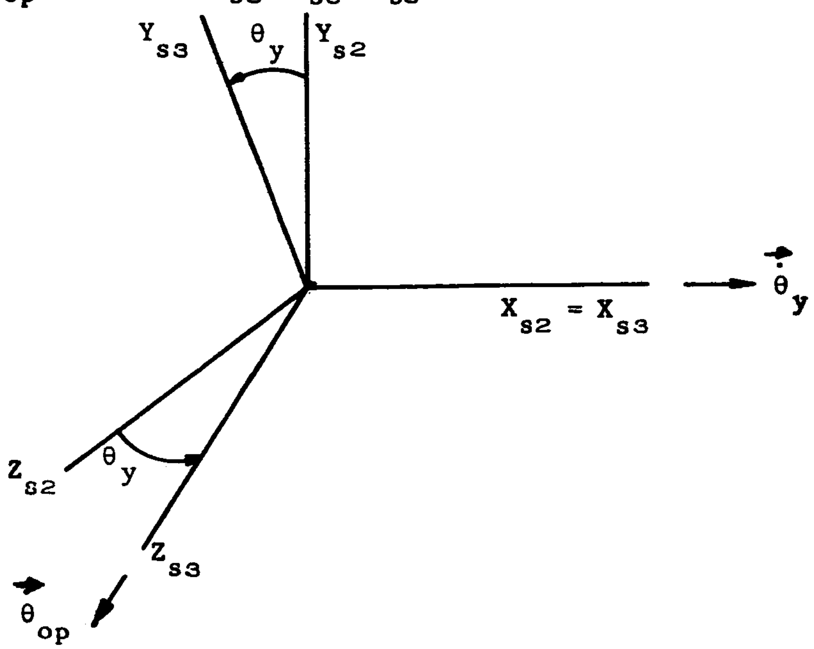
$\dot{\theta}_y$ is in the X_{s2}, Y_{s2}, Z_{s2} coordinate system.



The components of $\dot{\theta}_y$ in the vehicle coordinate system are

$$\begin{bmatrix} \dot{\phi}_{x\dot{\theta}_y} \\ \dot{\phi}_{y\dot{\theta}_y} \\ \dot{\phi}_{Z\dot{\theta}_y} \end{bmatrix} = A_4 \begin{bmatrix} \dot{\theta}_y \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

$\dot{\theta}_{op}$ is in the X_{s3}, Y_{s3}, Z_{s3} coordinate system.



The remaining rotation is about the Z_{s3} axis. This carries the X_{s3}, Y_{s3}, Z_{s3} coordinate system into the vehicle coordinate system.

$$\begin{bmatrix} \dot{\phi}_{x\dot{\theta}_{op}} \\ \dot{\phi}_{y\dot{\theta}_{op}} \\ \dot{\phi}_{Z\dot{\theta}_{op}} \end{bmatrix} = A_4 \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{op} \end{bmatrix} \tag{5}$$

$\dot{\theta}_{op}$ is along the same axis as $\dot{\phi}_z$. There are no components of $\dot{\theta}_{op}$ along the same axes as $\dot{\phi}_x$ and $\dot{\phi}_y$.

The resultant angular velocity about each axis ($\dot{\phi}_x, \dot{\phi}_y, \dot{\phi}_z$) is the sum of the respective components as a result of $\dot{\theta}_r$, $\dot{\theta}_{pl}$, $\dot{\theta}_y$, and $\dot{\theta}_{op}$. The equations become

$$\begin{aligned} \dot{\phi}_x &= b_{11}\dot{\theta}_y + b_{12}\dot{\theta}_r + b_{13}\dot{\theta}_{pl} \\ \dot{\phi}_y &= b_{21}\dot{\theta}_y + b_{22}\dot{\theta}_r + b_{23}\dot{\theta}_{pl} \\ \dot{\phi}_Z - \dot{\theta}_{op} &= b_{32}\dot{\theta}_r + b_{33}\dot{\theta}_{pl} \end{aligned} \tag{6}$$

$\dot{\theta}_{op} = \dot{\chi}_z$, which is a known angular rate,

where

$b_{11} = \cos \theta_{op}$

$b_{12} = \cos \theta_{op} \sin \theta_{pl} + \cos \theta_{pl} \cos \theta_y \sin \theta_{op}$

$b_{13} = \sin \theta_{op} \sin \theta_y$

$b_{21} = -\sin \theta_{op}$

$b_{22} = -\sin \theta_{op} \sin \theta_{pl} + \cos \theta_{pl} \cos \theta_y \cos \theta_{op}$

$b_{23} = \cos \theta_{op} \sin \theta_y$

and

$$a_{11} = \cos\theta_{op} - \sin\theta_{op} \tan\theta_{p1} \cos\theta_y$$

$$a_{12} = -(\sin\theta_{op} + \cos\theta_{op} \tan\theta_{p1} \cos\theta_y')$$

$$a_{13} = \sin\theta_y \tan\theta_{p1}$$

$$a_{21} = \frac{\sin\theta_{op} \cos\theta_y}{\cos\theta_{p1}}$$

$$a_{22} = \frac{\cos\theta_{op} \cos\theta_y}{\cos\theta_{p1}}$$

$$a_{23} = -\frac{\sin\theta_y}{\cos\theta_{p1}}$$

$$a_{31} = \sin\theta_{op} \sin\theta_y$$

$$a_{32} = \cos\theta_{op} \sin\theta_y$$

$$a_{33} = \cos\theta_y$$

Equation 7 may be solved for θ_y, θ_r, and θ_{p1} by either a digital or analog computer. This is an advantage because it will not restrict the choice of computer for simulating the vehicle.

These equations have been checked by programing equations of motion for a 7090 computer. The basic flow diagram for these equations is the same as FIGURE 1.

The platform resolver chain in FIGURE 1 is simulated by substituting θ_{op} and the solutions, θ_r, θ_{p1}, and θ_y, of these equations into the matrix product of equation 1. This product, $A_4\ A_3\ A_2\ A_1$, is the transformation matrix between the space-fixed coordinate system and the vehicle coordinate system.

This matrix, $A_4\ A_3\ A_2\ A_1$, is checked quantitatively by a parallel computation of the gimbal angle equations and the set of equations stated in the Appendix. The nine elements of the matrix, $A_4\ A_3\ A_2\ A_1$, should be equal to the respective elements of the matrix formed by the solution of the equations in the Appendix. These elements agree.

$$b_{31} = 0$$

$$b_{32} = -\cos \theta_{p1} \sin \theta_y$$

$$b_{33} = \cos \theta_y$$

$\dot{\phi}_x$, $\dot{\phi}_y$, and $\dot{\phi}_z$ are known from the equations of motion. The equations may be solved explicitly for $\dot{\theta}_y$, $\dot{\theta}_{p1}$, and $\dot{\theta}_r$, provided the determinant of the coefficients is non-zero. Let the matrix of the coefficients be B. Then,

$$|B| \neq 0$$

is the requirement for a solution to exist. Evaluation of $|B|$ yields

$$|B| = \cos \theta_{p1}.$$

This is zero for $\theta_{p1} = \frac{\pi}{2}$. However, the design of the platform restricts θ_{p1} to

$$-20° \leq \theta_{p1} \leq +20°$$

Thus, the singular points are avoided. The solutions of equation (6) for $\dot{\theta}_y$, $\dot{\theta}_{p1}$, and $\dot{\theta}_r$ yield

$$\dot{\theta}_y = a_{11} \dot{\phi}_x + a_{12} \dot{\phi}_y + a_{13} (\dot{\phi}_Z - \dot{\theta}_{op}) \tag{7}$$

$$\dot{\theta}_r = a_{21}\dot{\phi}_x + a_{22}\dot{\phi}_y + a_{23} (\dot{\phi}_Z - \dot{\theta}_{op})$$

$$\dot{\theta}_{p1} = a_{31}\dot{\phi}_x + a_{32}\dot{\phi}_y + a_{33} (\dot{\phi}_Z - \dot{\theta}_{op})$$

At $t = t_o$, the initial conditions are

$$\theta_y = \theta_y(t_o) \qquad \dot{\phi}_x = \dot{\phi}_x(t_o)$$

$$\theta_r = \theta_y(t_o) \qquad \dot{\phi}_y = \dot{\phi}_y(t_o)$$

$$\theta_{p1} = \theta_{p1}(t_o) \qquad \dot{\phi}_z = \dot{\phi}_z(t_o)$$

$$\dot{\theta}_{op} = \dot{X}_z(t_o)$$

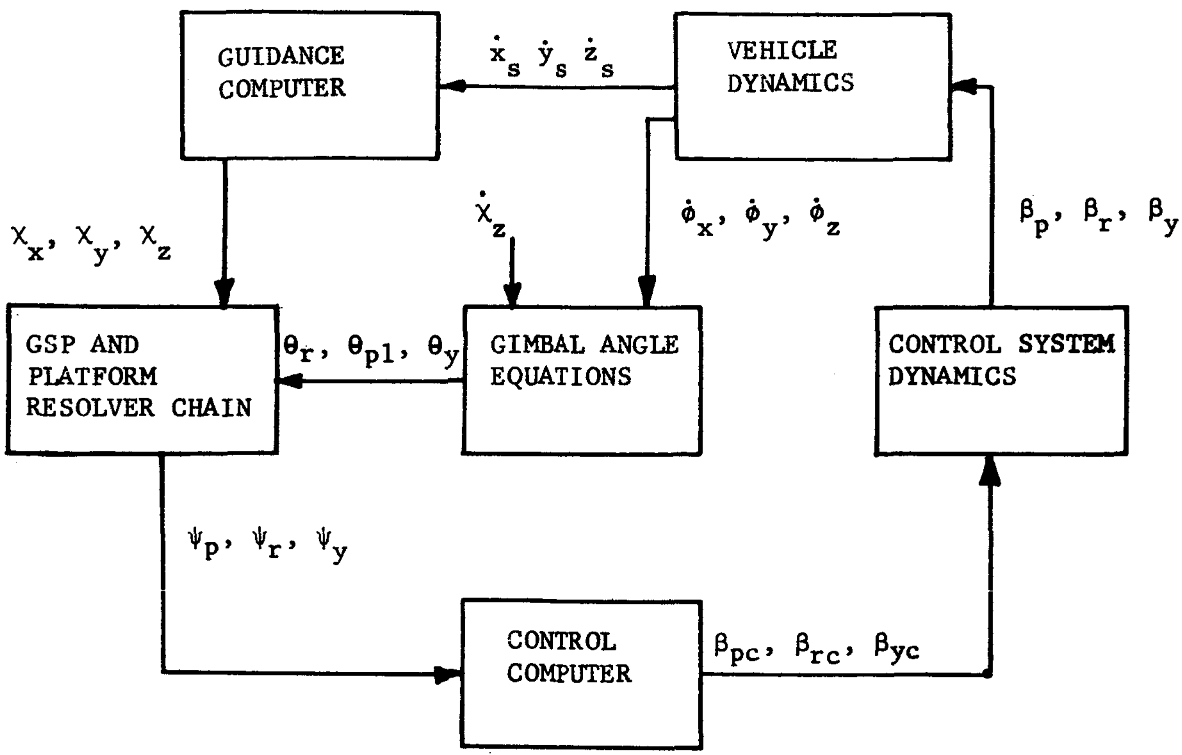


FIGURE 1. SIMPLE BLOCK DIAGRAM OF SATURN GUIDANCE AND CONTROL SIMULATION.

SECTION IV. CONCLUSION

This simulation of the platform gimbal angles offers numerous advantages and possibilities for study of the platform and its use in the guidance and control system.

APPENDIX

TRANSFORMATION MATRIX EQUATIONS

The following equations were used to check the gimbal angle equations. The derivations of these equations are adequately presented in the literature. Consequently, they are only stated here.

$$\dot{d}_{11} = d_{12}\dot{\phi}_Z - d_{13}\dot{\phi}_y$$

$$\dot{d}_{12} = d_{13}\dot{\phi}x - d_{11}\dot{\phi}_Z$$

$$\dot{d}_{13} = d_{11}\dot{\phi}_y - d_{12}\dot{\phi}_x$$

$$\dot{d}_{21} = d_{22}\dot{\phi}_Z - d_{23}\dot{\phi}_y$$

$$\dot{d}_{22} = d_{23}\dot{\phi}_x - d_{21}\dot{\phi}_Z$$

$$\dot{d}_{23} = d_{21}\dot{\phi}_y - d_{22}\dot{\phi}_x$$

$$\dot{d}_{31} = d_{32}\dot{\phi}_Z - d_{33}\dot{\phi}_y$$

$$\dot{d}_{32} = d_{33}\dot{\phi}_x - d_{31}\dot{\phi}_Z$$

$$\dot{d}_{33} = d_{31}\dot{\phi}_y - d_{32}\dot{\phi}_x$$

and initial conditions.

The transformation matrix given by the solution of these equations transforms from vehicle coordinates to space-fixed coordinates. It is

$$\begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}$$

BIBLIOGRAPHY

1. Kolk, W. Richard, "Modern Flight Dynamics," Prentice-Hall, 1961.

2. Moore, R. L. and Thomason, H. E., "Gimbal Geometry and Attitude Sensing of the ST-124 Stabilized Platform," NASA TN D-1118, May 1962.

APPROVAL MTP-ASTR-N-63-9

DYNAMIC SIMULATION OF ST-124 PLATFORM

By

C. D. Carlile

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

for G. G. Gassaway

F. B. Moore
Chief, Guidance and Control Systems Branch

W. Haeussermann

W. Haeussermann
Director, Astrionics Division

DISTRIBUTION

M-DIR
  Dr. von Braun

M-ASTR-DIR
  Dr. Haeussermann
  Miss Flowers

M-ASTR-TSS
  Mr. L. Richard

M-ASTR-G
  Mr. Thomason

M-ASTR-N
  Mr. Moore
  Mr. Gassaway
  Mr. Wood (10)
  Mr. McMahen
  Mr. Brooks
  Mr. Carlile (10)
  Mr. Winkler
  Mr. Thacker
  Mr. Brown
  Branch File
  Record Copy

M-ASTR-ADM
  Reference File

M-MS-IP

M-MS-IPL (8)

M-HME-P

M-MS-H

M-PAT

Scientific and Technical Information
  Facility (2)
Attn: NASA Representative (SAK-RKT)
P. O. Box 5700
Bethesda, Maryland